Topics in Undergraduate Math, Analysis of Fractals, MATH300, Fall 2014

Meets: Monday-Wednesday-Friday, 10-10:50 PM (Location, to be announced)

Description: To indicate the *mass* of a set, one makes an initial decision concerning which notion of mass to use. For a finite set, the mass would be simply the *number of points* in the set. For a curve, one would choose the *length* of the curve and, for a surface, the *area* of the surface, or for a region in space, the *volume* of the region. These four distinct notions of mass are examples of *Hausdorff measures* of dimension $0, 1, 2$ and 3. Remarkably, one can define and work with a specific notion of *d dimensional Hausdorff measure* of a set where d is any (not necessarily integer) positive number or 0. This notion has a mathematically precise (and natural) definition which will be developed at the beginning of the course. An interesting example will be the standard *Cantor set*. This set is obtained, starting with $[0, 1]$, by an infinite process of removing the middle-thirds of intervals at each step. We will see how this Cantor set has finite positive Hausdorff measure exactly in the dimension $\frac{\log 2}{\log 3}$.

We will go on to study many other specific examples of such sets which have a finite positive Hausdorff measure in some non-integer dimension. These sets are popularly called *fractals* and their construction often involves some repeated dynamical process. Over the last 30 years, fractals have gained increasing exposure in the popular mathematical and applied science literature. The fact that fractals may be generated by some repeated (or partially random) process has, with the growth of high-speed computing allowed more examples to be studied closely. Similarly rapidly developing computer graphics has led to better imaging of lower dimensional fractals that lie in the plane or in 3 space. Numerous applied fields have found it useful to model phenomena with non-integer sets: e.g. the distribution of mass in the universe, the networks of the circulatory or nerve systems, the jagged surface of asteroid, or even a rugged coastline. Although the popular literature on fractals often falls short on precise mathematical statements, we will in this course try to verify carefully many interesting mathematical facts. Part of the course will involve training and practice in writing clear, cogent complete mathematical proofs about the fractals we study.

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Textbook:
K. Falconer, *Fractal Geometry, Mathematical Foundations and Applications* J.Wiley, 2nd Ed.

Prerequisites: Math 102 or equivalent.

Requirements: We anticipate having weekly homework, a single midterm, and a final with the grade being determined by the percentages:
midterm 20%, final 25%, homework 50%, class participation 5% .

Accommodations: If you have a (documented) disability and you would like an accommodation, please speak with me during the first two weeks of class. Any such information will of course remain confidential, and you certainly do not need to tell us any details beyond those we should expect to be hearing from the Office of Disability Support Services describing the accommodations you will be requiring. You'll also have to file forms with Disability Support Services in the Ley Student Center in order to have those accommodations implemented.